ROUND I

1. The volume of a large cube is 64 cubic inches. A new shape is formed by removing a small cube from one corner of the large cube. What is the surface area of new shape?

Answer: 96 in$^2$

2. Find all solutions of the equation $|x - 2| = |2x^2 - 3x - 2|$.

Answer: $-1, 0, 2$

3. Suppose that $a$ and $b$ are nonzero real numbers, and that the equation $x^2 + ax + b = 0$ has solutions $a$ and $b$. What is the pair $(a, b)$?

Answer: $(1, -2)$

4. Suppose $f(x)$ is a function and $f(-1), f(0), f(1)$ are distinct real numbers. If $f(x^3) = (f(x))^3$ for all $x$, find $f(0) + f(1) + f(-1)$.

Answer: 0

5. If $\log_a x = 2$ and $\log_a y = 3$, what is the value of $\log_a \sqrt[3]{ax} \sqrt[3]{y}$?

Answer: $\frac{1}{5}$

6. For how many positive integers $n$ is $n^2 + 2n - 3$ prime?

Answer: One

7. How many distinct permutations are there of 3 letters selected from the letters in SYZYGY?

Answer: 34

8. Suppose $x$ is a complex number such that $x^2 + x + 1 = 0$. What is the value of $x^3$?

Answer: 1
ROUND II

1. A Rubik’s cube consists of 27 solid cubes, each of which has a side length of 1 inch. A new solid is formed by removing one cube from the center of each face of the Rubik’s cube. What is the surface area of new solid?

Answer: 78 in²

2. If \( p \) and \( q \) are the roots of \( x^3 - 2008x + 2 = 0 \), what is the value of \( \frac{1}{p} + \frac{1}{q} \)?

Answer: 1004

3. How many sets are there if each set contains three distinct numbers selected from the set \( \{0, 1, 2, 3, 4, 5\} \) so that their sum is an even number greater than 4?

Answer: 9

4. For what value of \( k \) does the equation \((x-1)(x-5) = x(x-k)\) have no solution for \( x \)?

Answer: \( k = 6 \)

5. If \( N = \sqrt{25^{32}16^{19}} \) in decimal representation, what is the sum of the digits of \( N \)?

Answer: 10

6. For what range of real numbers \( a \) is \( \log_2(ax-1) \) positive for \( 1 \leq x \leq 2 \)?

Answer: \( a > 2 \)

7. If \( \sin \left(\frac{\theta}{2}\right) + \cos \left(\frac{\theta}{2}\right) = 0.8 \), what is the value of \( \sin \theta \)?

Answer: −0.36
8. Circles of radii 3, 2, and 1 are externally tangent and are circumscribed by a fourth circle. Find the area of the shaded region.

Answer: $22\pi$