CSU High School Mathematics Tournament
March 6th, 2004

Ciphering Competition
Find the real solutions $x$ of the equation

$$\frac{1}{1 + \frac{1}{1 + \frac{1}{x}}} = x .$$

**ANSWER:** $\pm \frac{\sqrt{2}}{2}$
If $\sin(\alpha) = \frac{1}{3}$ what is the numerical value of $\cos(2\alpha)$?

ANSWER: $\frac{7}{9}$
In the square $ABCD$ the point $P$ on $AB$ satisfies $AP = 2PB$ and the point $Q$ is the midpoint of the side $CD$. Find the ratio $\frac{\text{Area}(APQD)}{\text{Area}(PBCQ)}$.

**ANSWER:** \( \frac{7}{5} \)
One picks two whole numbers at random. What is the probability that their product is even?

**ANSWER:** \( \frac{7}{8} \)
Two positive integers $x$ and $y$ satisfy $x + y = xy$. What is $x + y$?

ANSWER: 4
A bee flies inside of a cubic box with side lengths 10 meters from one corner directly to the center of a non-adjacent face. How many meters does it fly?

\[ \text{ANSWER: } 5\sqrt{6} \text{ or } \sqrt{120} \]
Solve for real $x$:

$$2^{2x} + 2^x = 30.$$ 

**ANSWER:** \( \frac{\ln 5}{\ln 2} \) or \( \log_2 5 \)
Determine if the following statement is true or false: “For all nonzero integers \(x\) and \(y\) either \(2x = 3y + 7\) or \(x^2 + y^2 \geq 2\).”

**ANSWER:** true
SCHOOL NAME: __
SCHOOL NUMBER:

Solve the equation

\[ 1 + x[1 + x(1 + x)] = \frac{1}{x - 1} . \]

ANSWER: ± \sqrt{2} and ± \sqrt{2}i
What is the smallest integer value of $n$ such that $n \cos \left( \frac{\pi}{6} \right) > 30$? (angles are measured in radians)

**Answer:** 35
The area of an isosceles right triangle $\triangle ABC$ ( $m\angle BAC = 90^\circ$) is divided into two equal parts by an arc of a circle centered at $A$ and of radius $r$. Find $r$ in terms of $AB$.

**ANSWER:** $\frac{AB}{\sqrt{\pi}}$
A whole number is chosen at random from the set \{1, 2, 3, \ldots, 2004\}. What are the chances that the number is divisible by 167?

**Answer:** \(\frac{1}{167}\)
Two positive integers $x$ and $y$ satisfy the equation $x^2 - xy = 2$. What is the value of $y$?

ANSWER: 1
What is the volume (in cubic meters) of the right circular cone in the figure below whose base radius is 3 meters and slide height is 5 meters?

\[ \text{ANSWER: } 12\pi \]
SCHOOL NAME: __
SCHOOL NUMBER: 

Solve for the positive $x$ the logarithmic equation

$$\ln x + \ln(x + 20) = \ln 3 + 2 \ln 10.$$ 

**ANSWER:** 10
ROUND 2, QUESTION 8

SCHOOL NAME: __
SCHOOL NUMBER:

Given $M = (1+\sqrt{2})^{2004}+(1-\sqrt{2})^{2004}$

your answer to this problem should be $A$ if $M$ is a whole number or $B$ if $M$ is an irrational number.

What is your answer?

ANSWER: $A$
SCHOOL NAME: __
SCHOOL NUMBER:

Solve the system of equations:

\[
\begin{align*}
2x + y + z &= 3 \\
x + 2y + z &= 7 \\
x + y + 2z &= 6
\end{align*}
\]

ANSWER: \(x = -1, y = 3, z = 2\)
How many different ordered pairs \((x, y)\) of real numbers satisfy the equation

\[ x^2 + y^2 + 9 = xy + 3x + 3y \]?

**ANSWER:** One
A fair coin is to be tossed 4 times. What is the probability that more of the tosses will result in heads than will result in tails?

**ANSWER:** $\frac{5}{16}$