TWENTY-THIRD ANNUAL COLUMBUS INVITATIONAL MATHEMATICS TOURNAMENT

Sponsored by

THE COLUMBUS STATE UNIVERSITY

DEPARTMENT OF MATHEMATICS

March 1, 1997

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The Columbus State University Mathematics faculty welcome you to this year's tournament and to our campus. We hope that you will be successful on this test.

INSTRUCTIONS: This is a 90-minute, 50-problem, multiple-choice examination. There are five (5) possible responses to each question. You are to select the one (1) "best" answer to each. You may mark on the test booklet and on the paper provided to you. If you need more paper or an extra pencil, let one of the monitors know. When you are sure of an answer, circle the choice you have made on the test booklet. After you have worked all of the problems that you can work, carefully transfer your answers to the score sheet. Darken completely the blank corresponding to the letter of your response to each question. Mark your answers boldly with a No. 2 pencil. If you must change an answer, completely erase your first choice and then record the new answer. Incomplete erasures and multiple marks for any question will be scored as an incorrect response. The examination will be scored on the basis of +12 for each correct answer, -3 for each incorrect selection and 0 for each omitted item. Each student will be given an initial score of +200.

Pre-selected problems will be used as tie-breakers for individual awards. These problems, in the order in which they will be examined, are: 47, 49, 48, 44, 46 and 50.

Throughout this exam, AB will denote the line segment from vertex A to vertex B. $\overline{AB}$ will denote the length of AB. All pre-drawn geometric figures are not necessarily drawn to scale.

Review and check your score sheet carefully. Your student identification number and your school number must be encoded correctly on your score sheet.

When you complete your test, bring your pencil, scratch paper and answer sheet to the Test Monitor. You may leave the room after you have handed in your answer sheet. Please leave quietly so as not to disturb the other contestants. Please do not congregate outside the doors to the testing area. You may keep your copy of the test. Your sponsor will have a copy of the answers.

DO NOT OPEN THIS TEST UNTIL YOU ARE INSTRUCTED TO DO SO

1. Simplify the following: $\dfrac{(2^3x^{-3}y^{-2}z)^{-2}}{(2^{-3}x^2y^{-1}z\ )^2}$

a) $\dfrac{4x^2y^6}{z^4}$ b) $\dfrac{x^2y^6}{4}$ c) $\dfrac{x^2}{1024y^2}$ d) $\dfrac{2}{x^9y^2z^4}$ e) $\dfrac{x^2y^6}{4z^4}$

2. Simplify $\dfrac{6a}{4-3a}+\dfrac{8}{3a-4}$.

a) -2 b) $\dfrac{6a+8}{3a-4}$ c) $\dfrac{9a-4}{12-3a}$ d) $3a-4$ e) $-\dfrac{6a+8}{3a-4}$

3. Find the product of the solutions to the following system of equations:

$$\begin{cases} 3x+4y=-6 \\ 7x+5y=-1 \end{cases}$$

a) -7 b) 6 c) -6 d) -28 e) -5

4. Assume that r and s are positive and solve for d: $\dfrac{d}{r}+\dfrac{d}{s}=t$

a) $\dfrac{t}{s+r}$ b) $\dfrac{t(r+s)}{2}$ c) $\dfrac{rst}{2}$ d) $t(s+r)$ e) $\dfrac{rst}{r+s}$

5. Find the slope of the line that contains the points (1,-5) and (-1,-9).

a) $\dfrac{1}{3}$ b) $\dfrac{1}{2}$ c) 1 d) 2 e) 3

6. At noon, Tom left point A walking due north. An hour later, Dick left point A walking due east. Both boys walked at 4 miles per hour and both carried walkie-talkies with a range of 8 miles. At approximately what time did they lose contact with each other?

a) 1:25 p.m. b) 1:30 p.m. c) 1:49 p.m. d) 2:00 p.m. e) 3:36 p.m.

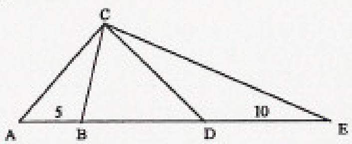
7. Find the product of all solutions to $3x^2 + 4x - 2 = 0$.

a) $\frac{2}{3}$ b) $\frac{-2}{3}$ c) $\frac{4}{3}$ d) $\frac{-4}{3}$ e) -24

8. Find the radius of the circle $x^2 + 3x + y^2 - 4y = 3$.

a) $\sqrt{37}$ b) $\sqrt{3}$ c) $\frac{\sqrt{34}}{2}$ d) $\frac{\sqrt{37}}{2}$ e) $\frac{\sqrt{57}}{2}$

9. In the figure below, the area of ΔABC is 40 and the area of ΔACE is 188. Find the length of segment $\overline{BD}$.



a) $\frac{17}{2}$ b) 10 c) 8 d) $8\sqrt{2}$ e) $5\sqrt{2}$

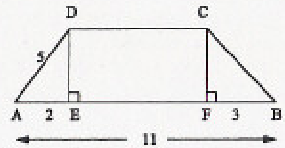
10. Now that she is over seventy, my great-aunt insists on demonstrating her youth on her birthday. Last year she hiked into the country at a rate of 3mph and bicycled back at 24 mph. How far into the country did she go assuming the entire trip took exactly 6 hours?

a) 10 miles b) 16 miles c) 14 miles d) 12 miles e) 6 miles

11. If 20 ounces of dough are used to make a pizza 16 inches in diameter, how many ounces are needed to make a pizza of the same thickness that is 20 inches in diameter?

a) 31.25 b) 25 c) 24 d) 35 e) 36

12. Find the area of trapezoid ABCD.

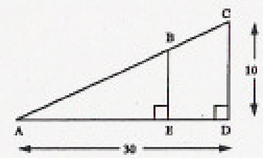


a) $\frac{11\sqrt{21}}{2}$ b) $11\sqrt{29}$ c) $8\sqrt{21}$ d) $\frac{17\sqrt{29}}{2}$ e) $\frac{17\sqrt{21}}{2}$

13. The polynomial $P(x)$ evaluated at $x = 4$ is 8. If $P(x) = Q(x)(x-4) + r$ for some polynomial $Q(x)$, find r.

a) 2 b) 4 c) 8 d) 16 e) 32

14. The area of ΔABE is one-half the area of ΔACD. Find the product of $\overline{AE}$ and $\overline{BE}$.



a) 300 b) $300 - 15\sqrt{2}$ c) 150 d) 75 e) $100 - 5\sqrt{2}$

15. Solve for x if a>0 and $a^{x+1} - a^2 = a$.

a) $\log_a(a^2 + a - 1)$ b) $\log_a(a^2 + a)$ c) $\log_a(a+1)$

d) 3 e) 2

16. Let A = { a, b, c } and B = { 1, 2, 3, 4, 5 }. Which set(s) of ordered pairs represent a function from set A to set B ?

(i) { (a,2), (b,3), (c,4) } (ii) { (a,4), (b,5) }

(iii) { (a,1), (b,1), (c,1) } (iv) { (a,1), (a,2), (b,3), (b,4), (c,5) }

a) (i) only b) (ii) and (iv) only c) (ii), (iii) and (iv) only

d) (i) and (iii) only e) (iv) only

17. A car radiator contains 10 quarts of 30% antifreeze solution. How many quarts will have to be replaced with pure antifreeze if the resulting solution is to be 50% antifreeze.

a) 3.143 b) 2.857 c) 3.500 d) 2.000 e) 3.000

18. A group of n people decide to buy a $36,000 minibus for a charitable organization. Each person will pay an equal share of the cost. If three additional people were to join the group, the cost per person would decrease by $1000. Find n.

a) 12 b) 6 c) 3 d) 9 e) 10

19. The stopping distance of an automobile is directly proportional to the square of its speed. A car required 75 feet to stop when its speed was 30 miles per hour. Estimate the stopping distance if the brakes are applied when the car is traveling at 50 miles per hour.

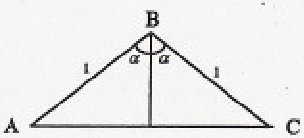
a) 125 feet b) 345 feet c) 208.33 feet

d) 187.34 feet e) 250.75 feet

20. A class of 20 students had an average of 84 on an algebra test . Another algebra class consisting of 25 students had an average of 66 on the same test. What is the average test grade for the two algebra classes combined?

a) 71 b) 72 c) 73 d) 75 e) 74

21. Find the area of the triangle ABC below.



a) $\cos 2\alpha$ b) $\sin 2\alpha$ c) $2\sin\alpha$ d) $\sin\alpha\ (1-\sin\alpha)$ e) $\dfrac{\sin 2\alpha}{2}$

22. Find the smaller of the angles for which the lines $y = 3x - 2$ and $y = -2x + 7$ intersect.

a) 39° b) 43° c) 45° d) 48° e) 53°

23. Find the area of the triangle bounded by the x-axis, $y = 3x + 2$ and $x = 2$.

a) $\dfrac{16}{3}$ b) $\dfrac{8}{3}$ c) $\dfrac{32}{3}$ d) 8 e) 4

24. If $0 < x < \dfrac{\pi}{2}$, the expression $\log(\cot x)$ is equivalent to which of the following?

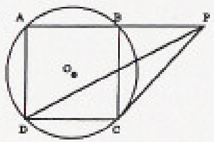
a) $\log(\cos x) + \log(\csc x)$ b) $\dfrac{1}{\log(\tan x)}$ c) $\log(\cos x) + \log(\sin x)$

d) $\dfrac{\log(\cos x)}{\log(\sin x)}$ e) $\log(\tan x)$

25. Find the following sum: $i^{0!} + i^{1!} + i^{2!} + \cdots + i^{100!}$

a) $95 + i$ b) $96 + i$ c) $97 + i$ d) $96 + 2i$ e) $95 + 2i$

26. In the diagram below ABCD is a square inscribed in a unit circle with center O. ABP is a line segment and PC is tangent to the circle. Find $\overline{PD}$.



a) $3\sqrt{2}$ b) $5\sqrt{2}$ c) $4\sqrt{2}$ d) $\sqrt{10}$ e) 3

27. Three rods are to be chosen from a box containing 5 rods measuring 15, 30, 40, 60 and 90 centimeters in length. What is the probability that they can be arranged in a triangle? (Assume that each rod is equally likely to be chosen.)

a) 2/5   b) 3/5   c) 4/5   d) 3/10   e) 1/2

28. Opening drain A will empty a swimming pool in 5 hours and opening both drain A and drain B will empty it in 2 hours. How long will it take to empty the pool if only drain B is opened?

a) 3 hours 20 minutes   b) 3 hours 40 minutes   c) 5 hours

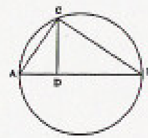
d) 3 hours 24 minutes   e) 3 hours 36 minutes

29. A homeowner needs four hours to mow his lawn. His son can mow the same lawn in six hours. How long will it take to mow the lawn if the father and son work together to mow half the lawn and the son finishes the job alone?

a) 5 hours 24 minutes   b) 4 hours 24 minutes   c) 5 hours 12 minutes

d) 4 hours 12 minutes   e) 5 hours 30 minutes

30. The diameter AB of the circle below has length 12 and CD, which is perpendicular to the diameter, has length 5. Find the length of AD, given that $\overline{AD} < 6$.



a) $6-\sqrt{11}$ b) $\frac{18-\sqrt{11}}{2}$ c) $\frac{12-\sqrt{11}}{2}$

d) $\frac{12+\sqrt{11}}{2}$ e) 6

31. How many three-digit natural numbers are there of the form stu, where $s > t > u$?

a) 648   b) 576   c) 504   d) 96   e) 120

32. Find the sum of all positive integers less than 10,000 that are both perfect squares and perfect cubes.

a) 1   b) 4   c) 2120   d) 12675   e) 4890

33. Find the coefficient of x^3 in $(1 - 2\sqrt{x})^8$.

a) 28   b) -1792   c) 1792   d) 64   e) -32

34. Of a group of red, blue and yellow marbles, all except 2 are red, all except 2 are blue and all except 2 are yellow. How many marbles are there?

a) 2   b) 3   c) 4   d) 5   e) 7

35. Consider the parabola $y = x^2$. Let T be the triangle with vertices on this parabola at $x = 1$, $x = 3$, and $x = -1$. Find the area of T.

a) 4 b) 8 c) 6 d) 5 e) 16

36. In the triangle ABC, if $A = 120^\circ$ and B is the smallest angle, then the ratio of $\overline{BC}$ to $\overline{AC}$ is

a) less than $\frac{\sqrt{2}}{2}$ b) less than $\frac{\sqrt{3}}{2}$ c) less than $\sqrt{2}$

d) less than $\sqrt{3}$ e) greater than $\sqrt{3}$

37. When a rubber ball is dropped, it rebounds to a height that is 75% of that from which it was dropped. From what height was the ball dropped if the total distance traveled was 286 feet at the time it struck the ground for the fourth time?

a) 2 b) 105 c) 60.2 d) 64 e) 90

38. Three married couples have purchased six seats in a row for musical comedy. How many different ways can they be seated if each couple must sit together.

a) 48 b) 6 c) 8 d) 9 e) 12

39. Bob and Tom, both 50% marksmen with a squirt gun, decide to fight a duel in which they alternate shots until one gets wet. What is the probability that the first one to shoot will win the duel?

a) 1/2 b) 3/4 c) 2/3 d) 13/20 e) 13/25

40. Find the sum of all the roots, both real and not real, of the equation $x^4 - 4x^3 + 3x^2 + 2x - 6 = 0$.

a) 4 b) 6 c) 3 d) $4 + 2i$ d) $4 - 2i$

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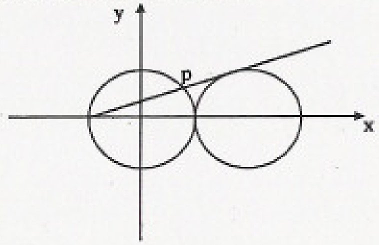
41. Determine the area (in square units) of the region which is the intersection of the regions $|x| + |y| \le 1$ and $y \ge |x|$.

a) 2 b) 1 c) 4 d) 0.5 e) 3

42. The figure below shows two abutting circles of radius 1, one centered at the origin, the other at (2, 0). Find the exact coordinates of p, the point where the line through (-1, 0) and tangent to the second circle meets the first circle.



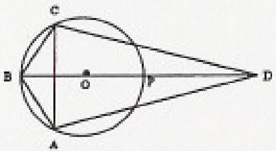
a) $(\frac{\sqrt{5}}{3}, \frac{2}{3})$ b) $(\frac{2}{3}, \frac{\sqrt{5}}{3})$ c) $(\frac{5}{9}, \frac{2\sqrt{14}}{9})$

d) $(\frac{7}{9}, \frac{4\sqrt{2}}{9})$ e) $(\frac{\sqrt{3}}{2}, \frac{1}{2})$

43. Let $z_k = \frac{k}{k+1}(\cos 10^\circ + i \sin 10^\circ)$. Find the exact value of the product $z_1 z_2 z_3 \cdots z_{179} z_{180}$.

a) $\frac{1}{180}$ b) $\frac{i}{180}$ c) $-\frac{1}{181}$ d) $\frac{1}{181}$ e) $\frac{i}{181}$

44. The unit circle below has center O and diameter $\overline{BP}$. $\overline{AC}$ is a perpendicular bisector of $\overline{OB}$. $\overline{BP}$ is extended to D so that $\overline{BP} = \overline{PD}$. Find the perimeter of the quadrilateral ABCD.



a) $2+5\sqrt{2}$ b) $1+5\sqrt{2}$ c) $1+2\sqrt{13}$ d) $3+2\sqrt{5}$ e) $2+2\sqrt{13}$

45. If $\cos\theta = \frac{1}{3}$ and $270^\circ < \theta < 360^\circ$, find $\cos\frac{\theta}{2}$.

a) $\frac{\sqrt{3}}{3}$ b) $-\frac{\sqrt{3}}{3}$ c) $\frac{\sqrt{3}}{2}$ d) $\frac{\sqrt{6}}{3}$ e) $-\frac{\sqrt{6}}{3}$

46. A local political organization is about to elect a president, a vice-president and a treasurer. Persons A, B and C have each decided to run for president. If not elected, they will run for vice-president. If again unsuccessful, they will run for treasurer. Persons D and E are going to run for vice-president and, if not successful, will run for treasurer. Person F will run for president only. Persons G and H are to run for treasurer only. How many possible outcomes are there for this election?

a) 64 b) 90 c) 140 d) 6720 e) 40320

47. Find the product of all coordinates of all points (x,y) that satisfy both $2x^2 + xy + 10y^2 = 22$ and $xy - 2y^2 + 6 = 0$.

a) $\frac{81}{49}$ b) $\frac{576}{25}$ c) $\frac{324}{49}$ d) $\frac{81}{25}$ e) $\frac{24}{49}$

48. If $\frac{df}{dx} = \frac{2x-1}{1-3x}$ and $g(x) = f(1-2x)$, find $\frac{dg}{dx}$.

a) $\frac{7x-3}{3x-1}$ b) $-\frac{7x-3}{3x-1}$ c) $\frac{(2x-1)^2}{(3x-1)^2}$

d) $-\frac{4x-1}{3x-1}$ e) $\frac{4x-1}{3x-1}$

49. Find the sum of all integral coordinates of the points (x, y) with $x > 0$, $y > 0$, and $x + y < 101$. That is, find $\sum_{2 \le x+y \le 100} (x+y)$, where x and y are positive integers.

a) 166650 b) 49500 c) 333300 d) 666600 e) 1010000

50. The area A of an ellipse of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with a and b positive is given by $A = \pi ab$. Find the area of the ellipse $x^2 - 2x + 2y^2 + 6y = \frac{1}{2}$.

a) $\frac{21\sqrt{2}\pi}{4}$ b) 6π c) $3\sqrt{2}\pi$ d) 18π e) $\frac{9\sqrt{2}\pi}{4}$