

ON THE STABILITY OF SOLUTIONS TO SOME ABSTRACT EVOLUTION EQUATIONS.

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ABSTRACT. The stability of the solution to the equation $\dot{u} = A(t)u + G(t, u) + f(t)$, $t \geq 0$, $u(0) = u_0$ is studied. Here $A(t)$ is a linear operator in a Hilbert space H and $G(t, u)$ is a nonlinear operator in H for any fixed $t \geq 0$. We assume that $\|G(t, u)\| \leq \alpha(t)\|u\|^p$, $p > 1$, and the spectrum of $A(t)$ lies in the half-plane $\operatorname{Re} \lambda \leq \gamma(t)$ where $\gamma(t)$ can take positive and negative values. We proved that the equilibrium solution $u(t) \equiv 0$ to the equation is Lyapunov stable under persistently acting perturbations $f(t)$ if $\sup_{t \geq 0} \int_0^t \gamma(\xi) d\xi < \infty$ and $\int_0^\infty \alpha(\xi) d\xi < \infty$. In addition, if $\int_0^t \gamma(\xi) d\xi \rightarrow -\infty$ as $t \rightarrow \infty$, then we proved that the equilibrium solution $u(t) \equiv 0$ is asymptotically stable under persistently acting perturbations $f(t)$. Sufficient conditions for the solution $u(t)$ to be bounded and for $\lim_{t \rightarrow \infty} u(t) = 0$ are proposed and justified.

The stability and global existence of solutions to some other equations are also considered.

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