Using Maple in Classes for Mathematics and Science Majors

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WHY TECHNOLOGY

CUPM Curriculum Guide on use of technology

The Mathematical Association of America's Committee on the Undergraduate Program in Mathematics (CUPM) made a number of recommendations in the 2004 CUPM Curriculum Guide to lead mathematics departments in designing curricula for their undergraduate students.

The guidelines underscore importance for departments of mathematical sciences

- to understand student needs, strengths and weaknesses, aspirations and career paths;
- to collaborate with colleagues from other departments on modifying and developing mathematics courses;
- to appropriately incorporate technology:

"At every level of the curriculum, some courses should incorporate activities that will help all students' progress in learning to use technology

- Appropriately and effectively as a tool for solving problems;
- As an aid to understanding mathematical ideas."

Lace and Windows

Pedagogical Considerations

1. **Emphasis on:**

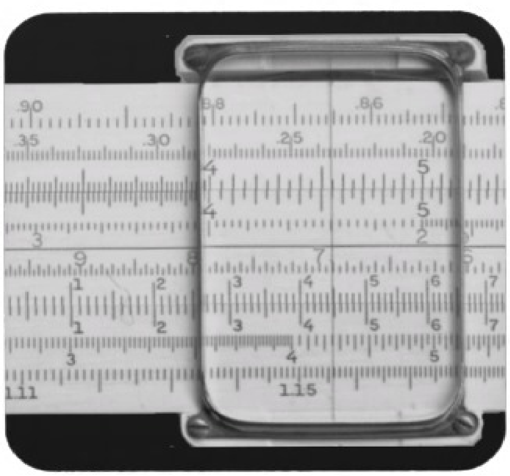
Concepts and applications vs arithmetic manipulations

2. **Why Maple:**

powerful computational tool + word processor =
easy means for keeping information, notes and computations in the same documents
symbolic computations
visualization
learning by doing
reasonably easy to use

3. **Modern Technology that is continuo developing and improving**

Slide Rule Coasters...



$with(Student[LinearAlgebra])$:

Implementation

Using Maple in Computer Lab

Calculus I, II, Linear Algebra at UofL, Linear Algebra Fall 2012

Lecture Outlines

For students to download and work with in class, library and at home, if possible.
Present examples as learn new concepts, make notes in class, have the work organized.

Sample Outline on Linear Independence

The definition below provided a different perspective on the homogeneous equation Ax=0 and the question of its non-trivial solutions:

Definition: An indexed set of vectors $\{v_1, v_2, ..., v_p\}$ in R^n is said to be **linearly independent** if the vector equation

$$x_1 v_1 + x_2 v_2 + ... + x_p v_p = 0$$

has only the trivial solution. The set $\{v_1, v_2, ..., v_p\}$ is said to be **linearly dependent** if there exist weights $\{c_1, ..., c_p\}$ not all zero, such that

$$c_1 v_1 + c_2 v_2 + ... + c_p v_p = 0.$$

Etc....

Example 1.

(a) Determine if the vectors given below are linearly independent:
$v_1 := \langle 1, 2, 3\rangle : v_2 := \langle 2, 4, 7\rangle : v_3 := \langle 3, 6, 10\rangle :$
(b) Find a linear relation if one exists.

Solution. (a) We need to determine if the equation $v_1 x_1 + v_2 x_2 + v_3 x_3 = 0$ has a non-trivial solution. Set
$A := \langle v_1 | v_2 | v_3 \rangle$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 7 & 10 \end{bmatrix} \qquad \textbf{(2.2.1.1.1)}$$

$\xrightarrow{\text{row-reduced form}}$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad \textbf{(2.2.1.1.2)}$$

Example 2.

For what values of h (IF ANY) is the set of vectors {v1, v2, v3} linearly dependent?
What are other problems equivalent to this one (in terms of systems, matrix equations...)?

$u_1 := \langle 1, -2, -4 \rangle : u_2 := \langle -3, 7, 6 \rangle : u_3 := \langle 2, 1, h \rangle :$

$B := \langle u_1 | u_2 | u_3 \rangle$

$$\begin{bmatrix} 1 & -3 & 2 \\ -2 & 7 & 1 \\ -4 & 6 & h \end{bmatrix} \qquad \textbf{(2.2.1.2.1)}$$

$\xrightarrow{\text{row-reduced form}}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \textbf{(2.2.1.2.2)}$$

An important comment: students are encouraged to use Student[LinearAlgebra] package.
The computation above is the output for a specific command used with that package.

$GaussianElimination(B)$

$$\begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & h+38 \end{bmatrix} \qquad \textbf{(2.2.1.2.3)}$$

Projects

Required to be completed in Maple; to explain the work coherently.

Sample: Project 2

$with(Student[LinearAlgebra]):$

MAPLE PROJECT 2

Instructions

1. Please, save this document under your name or names, e.g. Levi Project 2.
2. List the names of all the members of your group; a group submits one document.
3. If there is an issue with participation of a member of the group, attach a written document signed by all the rest of the members of the group specifying the issue. Else all the members of the group will receive the same grade.
4. Do all the work in this document, and type your replies in Maple.
5. Answer each problem in the section in Maple that it was defined.
6. Submit your Maple file to me by e-mail as an attachment.
7. The subsection below provides a limited guidance on Maple editing. Use Maple help menus liberally.

How to write in Maple

If Maple opens in math mode, press F5, to shift to text. F5 is a toggle; now I want math: $f(x) = x^2 + 3\cdot x + 5$. Note that this does NOT execute when I press Enter in text mode, unless I am in the math mode block: $2 + 3$

$$5 \qquad \textbf{(2.3.1.1.1.1)}$$

Note that I am still in text mode after the executed statement. But ctrl-= will execute inline: $2 + 3 = 5$. Press F5 after the control - =, to resume text mode. $3\cdot 4 = 12$.

Note this toggling is like pressing the capsLock key: enter upper case mode and stay there, or *ctrl-i to enter italics mode,* ctrl-i will then exit italics. Other toggles: **ctrl-b toggles bold,** enter or exit.

Press F1 for the quick Help menu, F2 for the Quick Reference menu. DO THIS NOW and see the information available.

Problem 1: Markov Chain

A multinational company consists of three sub-companies: AsiaChip, AmeroGick, and AusiColor.
As the companies merged, 31% of funds was in AsiaChip, 46% of funds was in AmeroGick, and 23% was in AusiColor. Each year 1/2 of AmeroGick's funds stays in AmeroGick, and 1/4 goes to each of AsiaChip and AusiColor. For AsiaChip, 1/3 of their funds stays in AsiaChip, 1/3 goes to AmeroGick, and 1/3 goes to AusiColor. For AusiColor, 2/3 of their funds stays in AusiColor, and 1/3 goes to AmeroGick.

a) What is the amount of funds AsiaChip has after one year?
b) Suppose k years from the start the distribution of funds between the three sub-companies is $\langle As, Am, Au\rangle = \begin{bmatrix} As \\ Am \\ Au \end{bmatrix}$. What is the amount of funds AsiaChip has next year?

c) Find the matrix A s.t.

$$\begin{bmatrix} \text{AsiaChip} \\ \text{AmeroGick} \\ \text{AusiColor} \end{bmatrix}_{(k+1)st\ year} = A.\begin{bmatrix} \text{AsiaChip} \\ \text{AmeroGick} \\ \text{AusiColor} \end{bmatrix}_{kth\ year}$$

(Such a process is referred to us a Markov process; check out Section 4.9. Hint: each column of A should add up to 1. For extra-credit (2 pts) explain why it is so.)

d) Find the eigenvectors and eigenvalues of A.

e) Prove that A is diagonalizable, and let K be the diagonal matrix similar to A. Find K.

f) Calculate the limit of K^m as $m \to \infty$. Use limits of rational functions to prove your result.

g) Using part (f) above calculate the limit of A^m as $m \to \infty$.

h) Find the long term distribution of funds in the multinational company. JUSTIFY your results.

Problem 2 (Predator-Prey Model)

In a certain region, rats provide up to 90% of food for owls. Denote the owl and rat population at a time k by $x_k = \langle O_k, R_k \rangle$ where k is time in months, O_k is the number of owls in the region and R_k is the number of rats (in thousands) in the region.

Suppose that with no rats only 25% of the owls will survive each month, while if there is an unlimited supply of rats, the 0.3 R_k will make the owl population rise.

Also with no owls as predators, a population of rats will grow by 20% each month. So we may assume that $x_{k+1} = \langle \langle .25, -p \rangle | \langle 0.3, 1.2 \rangle \rangle .x_k = x_{k+1} = \begin{bmatrix} 0.25 & 0.3 \\ -p & 1.2 \end{bmatrix} .x_k$

where p is a positive parameter to be specified (1000p is the average amount of rats eaten by an owl in a month.) See also Example 1 in Section 5.6 in the textbook.

Here is some rationale for the model: we can think of

$O_{k+1} = \left(.25 + \frac{0.3 \cdot R_k}{O_k} \right) O_k$, that is for the population of owls to be the same, $0.25 + \frac{0.3 \cdot R_k}{O_k}$ has to be 1, and for it to increase, the term $0.25 + \frac{0.3 \cdot R_k}{O_k}$ has to be greater than 1. The ratio $\frac{0.3 \cdot R_k}{O_k}$ represents increase in the owl population due to food availability.

We can think of $R_{k+1} = -p \cdot Q_k + 1.2 \cdot R_k$. Here the term $-p \cdot Q_k$ represents the reduction in rat population due to owls, and the term $1.2 \cdot R_k$ represents the increase in rat population without the owls as predators.

a) Take p=0.113. Calculate what happens with the population of owls and rats in the long run.

Hint: Let $B := \langle\langle .25, -p\rangle | \langle 0.3, 1.2\rangle\rangle = \begin{bmatrix} 0.25 & 0.3 \\ -p & 1.2 \end{bmatrix}$. Find the eigenvalues and eigenvectors of B . Write out the solution x_k as a linear combination of eigenvectors of B. See what happens in the long run.

b) Find p such that both populations grow. Describe explicitly the rate growth for each population.

Hint: How many distinct eigenvalues does B have? How many distinct eigenvalues? What are their values relative to 1 (larger, smaller?) How does it relate to p?

c) Find p such that both populations become extinct. Justify your results and conclusions.

Students: feedback and performance

FALL 2012 Student Feedback

- 73.33% believed that university level Mathematics courses should be taught using technology (e.g. Maple, Excel, Mathematica, Graphing calculators, web calculators etc.)
- 100% of students used Maple, and 86.76% found it useful.

FALL 2012 Student Performance

Students had a choice to use Maple or other technology or none at all. The caveat: if miscalculations change the nature of the intended result of the problem, there is no credit.

Q: which students prefered to do computations by hand?

Grades: 25% A, 31.3% B, 31.3% C, 6.3% D, 6.3% F of the people in the class after the drop date.

Conclusion

Good learning, understanding of mathematical concepts, applications of math.
A lot of hard work on the side of each party.

Definitely worth it!
And it is the future....